Intersection between Microscopic and Macroscopic Abelian Dominance in the Confinement Physics of QCD

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ABSTRACT

We study abelian dominance for confinement in terms of the local gluon properties in the maximally abelian (MA) gauge, where the diagonal component of the gluon is maximized by the gauge transformation. We find microscopic abelian dominance on the link-variable for the whole region of β in the lattice QCD in the MA gauge. The off-diagonal angle variable, which is not constrained by the MA-gauge fixing condition, tends to be random besides the residual gauge degrees of freedom. Within the random-variable approximation for the off-diagonal angle variable, we analytically prove that off-diagonal gluon contribution $W^{\rm off}$ to the Wilson loop obeys the perimeter law in the MA gauge. The perimeter-law behavior of $W^{\rm off}$ is also confirmed using the lattice QCD simulation. This indicates macroscopic abelian dominance for the string tension.

1. Introduction

In the low-energy region of QCD, there appear interesting phenomena such as color confinement and chiral symmetry breaking reflecting the strong gauge-coupling. However, because of nonperturbative and nonabelian nature, these phenomena are difficult to treat analytically, and it is desired to extract the relevant degrees of freedom for description of infrared phenomena.

In 1974, Nambu proposed an idea that quark confinement can be interpreted using the dual version of the superconductivity.[1] In the superconductor, Cooper-pair condensation leads to the Meissner effect, and the magnetic flux is squeezed like a quasi-one-dimensional tube as the Abrikosov vortex. In this dual-superconductor picture for the QCD vacuum, the squeezing of the color-electric flux between quarks is realized by the dual Meissner effect as the result of condensation of color-magnetic monopoles. However, there are two following large gaps between QCD and the dual superconductor picture; 1) This picture is based on the abelian gauge theory, while QCD is a nonabelian gauge theory. 2) The dual-superconductor scenario requires condensation of magnetic monopoles as the key concept, while QCD does not have such a

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monopole as the elementary degrees of freedom. As the connection between QCD and the dual superconductor scenario, 't Hooft proposed the concept of the abelian gauge fixing[2] with assumption of abelian dominance for the infrared QCD. The abelian gauge fixing is defined so as to diagonalize a suitable gauge-dependent variable $\Phi[A_\mu(x)]$ and reduces QCD into an abelian gauge theory, where the off-diagonal element of the gluon field behaves as a charged matter field. Moreover, in the abelian gauge, color-magnetic monopoles appear as topological objects corresponding to the nontrivial homotopy group $\Pi_2(\mathrm{SU(N_c)}/\mathrm{U(1)}^{\mathrm{N_c}-1}) = \mathbf{Z}_\infty^{\mathrm{N_c}-1}$. If monopole condenses, the scenario of color confinement by the dual Meissner effect would be realized in QCD. In this paper, with the help of the lattice QCD simulation, we study intersection between abelian dominance of the gluon field (microscopic variable) and confinement force (macroscopic variable) as the theoretical basis of dual superconductor picture.

2. Microscopic Abelian Dominance in the Maximally Abelian Gauge

Abelian dominance on the confinement force have been investigated using the lattice QCD simulation in the maximally abelian (MA) gauge.[4] In terms of the link variable $U_\mu(s) \equiv U_\mu^0(s) + i\tau^a U_\mu^a(s)$, the MA gauge fixing is defined by maximizing $R \equiv \sum_{s,\mu} \mathrm{tr}\{U_\mu(s)\tau_3 U_\mu^\dagger(s)\tau_3\} = \sum_{s,\mu}\{(U_\mu^0(s))^2 + (U_\mu^3(s))^2 - (U_\mu^1(s))^2 - (U_\mu^2(s))^2\}$ through the gauge transformation. In the MA gauge, the off-diagonal components, U_μ^1 and U_μ^2, are forced to be small, and therefore the QCD system seems describable only by U(1)-like variables approximately. The MA gauge is a sort of the abelian gauge, because the MA gauge fixing diagonalizes $\Phi(s) \equiv \sum_{\mu,\pm} U_{\pm\mu}(s)\tau_3 U_{\pm\mu}^\dagger(s)$ with $U_{-\mu}(s) \equiv U_\mu^\dagger(s-\mu)$. In this section, we study abelian dominance on the link variable $U_\mu(s)$.

In the lattice formalism, the SU(2) link variable $U_\mu(\mathrm{s})$ is factorized as

$$U_\mu(s) = \begin{pmatrix} \cos\theta_\mu(s) & -\sin\theta_\mu(s)e^{-i\chi_\mu(s)} \\ \sin\theta_\mu(s)e^{i\chi_\mu(s)} & \cos\theta_\mu(s) \end{pmatrix} \begin{pmatrix} e^{i\theta_\mu^3(s)} & 0 \\ 0 & e^{-i\theta_\mu^3(s)} \end{pmatrix} \equiv M_\mu(s)u_\mu(s).$$

Here, the $\mathrm{U(1)}_3$ link-variable $u_\mu(s)$ corresponds to the diagonal gluon part and behaves as the abelian gauge filed in the MA gauge, while $M_\mu(s)$ corresponds to the off-diagonal gluon part.

In order to investigate abelian dominance on the link variable in the MA gauge, we define "abelian projection rate"[4] as $R_{\rm Abel} = \cos\theta_\mu(s) \in [0,1]$ with $0 \le \theta_\mu \le \frac{\pi}{2}$. For instance, the SU(2) link variable becomes completely diagonal if $\cos\theta = 1$, while it becomes off-diagonal if $\cos\theta = 0$. In Fig.1, we show local abelian projection rate $R_{\rm Abel}$ expressed by the arrow $(\sin\theta, \cos\theta)$ in a typical configuration of the lattice QCD. In the MA gauge, most of all SU(2) link variables become U(1)-like. For the quantitative argument, we show in Fig.2 the probability distribution $P(R_{\rm Abel})$ of the abelian projection rate $R_{\rm Abel}$. Without gauge fixing, one finds the average $\langle R_{\rm Abel}\rangle = \frac{2}{3}$. In the MA gauge, the off-diagonal component of the SU(2) link variable is forced to be reduced, and $R_{\rm Abel}$ approaches to unity; one obtains $\langle R_{\rm Abel}\rangle_{\rm MA} \simeq 0.93$ on 16^4 lattice with $\beta = 2.4$. Thus, we find *microscopic abelian dominance* on the link variable.

Fig.1 Local abelian projection rate $R_{\rm Abel} \equiv \cos\theta$ $(0 \le \theta_\mu \le \frac{\pi}{2})$ at β= 2.4 on 16^4 lattice without gauge fixing (left) and in MA gauge fixing (right). The arrow expresses $(\sin\theta, \cos\theta)$.

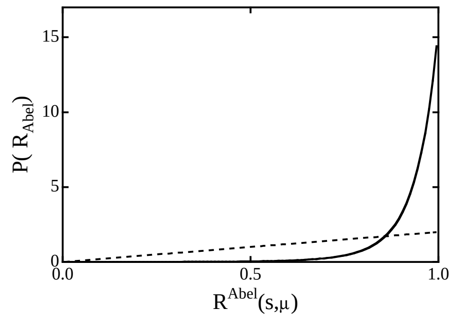

Fig.2 The probability distribution $P(R_{\rm Abel})$ of abelian projection rate $R_{\rm Abel}$ at $\beta = 2.4$ on 16^4 lattice in the MA gauge (solid curve) and without gauge fixing (dashed curve).

3. Semi-analytical Proof of Abelian Dominance for Confinement

In the MA gauge, the diagonal element $\cos\theta_\mu(s)$ in $M_\mu(s)$ is maximized by the gauge transformation. Then, the off-diagonal element $e^{i\chi_\mu(s)}\sin\theta_\mu(s)$ is forced to take a small value in the MA gauge, and therefore the approximate treatment on the off-diagonal element would be allowed in the MA gauge. Moreover, the angle variable $\chi_\mu(s)$ is not constrained by the MA gauge-fixing condition at all, and tends to take a random value[4] besides the residual $\mathrm{U}(1)_3$ gauge degrees of freedom. Hence, $\chi_\mu(s)$ in the MA gauge can be regarded as a *random angle variable* in a good approximation. In this section, we first investigate properties of $\chi_\mu(s)$ using the lattice QCD simulation, and then study the origin of abelian dominance on confinement.

We examine the randomness of $\chi_\mu(s)$ using the lattice QCD simulation in the MA gauge with $\mathrm{U}(1)_3$ Landau-gauge fixing.[4] We show in Fig.3 the probability distribution $P(\Delta\chi)$ of the correlation $\Delta\chi(s) \equiv {\rm mod}_\pi|\chi_\mu(s) - \chi_\mu(s+\hat\nu)| \in [0,\pi]$, which is the difference between two neighboring angle variables, at β=0, 1.0, 2.4, 3.0. In the strong-region as $\beta \le 1$, $\chi_\mu(s)$ behaves as a random variable, and there is no correlation between neighboring χ_μ. On the other hand, in the weak-coupling region, the smallness of $\sin\theta_\mu$ makes off-diagonal components more irrelevant in the MA gauge, which permits the approximate treatment on $\chi_\mu(s)$. Thus, we can take the random-variable approximation for $\chi_\mu(s)$ as a good approximation in the whole region of β in the MA gauge.

Next, let us consider the Wilson loop $\langle W_C[U_\mu(s)]\rangle \equiv \langle {\rm tr}\Pi_C U_\mu(s)\rangle$ in the MA gauge. In calculating $\langle W_C[U_\mu(s)]\rangle$, the expectation value of $e^{i\chi_\mu(s)}$ in $M_\mu(s)$ vanishes as $\langle e^{i\chi_\mu(s)}\rangle \simeq \int_0^{2\pi} d\chi_\mu(s)\exp\{i\chi_\mu(s)\} = 0$ within the random-variable approximation on $\chi_\mu(s)$. Then, the off-diagonal factor $M_\mu(s)$ appearing in the Wilson loop $W_C[U_\mu(s)]$ becomes a diagonal matrix, $U_\mu(s) \equiv M_\mu(s)u_\mu(s) \to \cos\theta_\mu(s)u_\mu(s)$.

Then, for the $I \times J$ rectangular C, the Wilson loop $W_C[U_\mu(s)] \equiv \langle {\rm tr}\Pi_{i=1}^L U_{\mu_i}(s_i)\rangle$ in the MA gauge is estimated as

$$\begin{aligned}\langle W_C[U_\mu(s)]\rangle &\simeq \langle {\rm tr}\Pi_{i=1}^L \cos\theta_{\mu_i}(s_i) u_{\mu_i}(s_i)\rangle_{\rm MA} = \langle \Pi_{i=1}^L \cos\theta_{\mu_i}(s_i)\cdot{\rm tr}\Pi_{j=1}^L u_{\mu_j}(s_j)\rangle_{\rm MA} \\ &\simeq \exp\{L\langle \ln(\cos\theta_\mu(s))\rangle_{\rm MA}\}\ \langle W_C[u_\mu(s)]\rangle_{\rm MA}, \qquad (1)\end{aligned}$$

where $L \equiv 2(I+J)$ denotes the perimeter length and $W_C[u_\mu(s)] \equiv \mathrm{tr}\Pi_{i=1}^{L} u_{\mu_i}(s_i)$ the abelian Wilson loop. Here, we have replaced $\sum_{i=1}^{L} \ln\{\cos(\theta_{\mu_i}(s_i)\}$ by its average $L\langle \ln\{\cos\theta_\mu(s)\}\rangle_{\rm MA}$ in a statistical sense. In this way, we derive a simple estimation as

$$W_C^{\rm off} \equiv \langle W_C[U_\mu(s)]\rangle / \langle W_C[u_\mu(s)]\rangle_{\rm MA} \simeq \exp\{L\langle \ln(\cos\theta_\mu(s))\rangle_{\rm MA}\} \tag{2}$$

for the contribution of the off-diagonal gluon element to the Wilson loop. From this analysis, $W_C^{\rm off}$ is expected to obey the *perimeter law* in the MA gauge for large loops, where the statistical treatment would be accurate.

In the lattice QCD, we find that $W_C^{\rm off}$ seems to obey the perimeter law for the Wilson loop with $I, J \geq 2$ in the MA gauge (Fig.4). We find also that the lattice data of $W_C^{\rm off}$ as the function of L are well reproduced by the above analytical estimation with microscopic information on $\cos\theta_\mu(s)$ as $\langle \ln\{\cos\theta_\mu(s)\}\rangle_{\rm MA} \simeq -0.082$ for $\beta = 2.4$.

Thus, the off-diagonal contribution $W_C^{\rm off}$ to the Wilson loop obeys the perimeter law in the MA gauge, and therefore the SU(2) string-tension $\sigma_{\rm SU(2)} \equiv -\lim_{I,J\to\infty} \frac{1}{IJ} \ln\langle W_{I\times J}[U_\mu(s)]\rangle$ coincides with to the abelian string-tension $\sigma_{\rm Abel}$,

$$\sigma_{\rm SU(2)} = -2\langle \ln\{\cos\theta_\mu(s)\}\rangle_{\rm MA} \frac{I+J}{IJ} + \sigma_{\rm Abel} \quad \stackrel{I,J\to\infty}{\longrightarrow} \quad \sigma_{\rm Abel}. \tag{3}$$

Thus, abelian dominance for the string tension, $\sigma_{\rm SU(2)} = \sigma_{\rm Abel}$, can be proved in the MA gauge by approximating the off-diagonal angle variable $\chi_\mu(s)$ as a random variable.

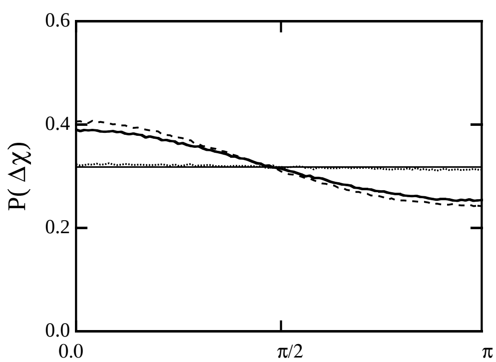

Fig.3 The probability distribution $P(\Delta\chi)$ of the correlation $\Delta\chi \equiv \mathrm{mod}_\pi(|\chi_\mu(s) - \chi_\mu(s+\hat\nu)|)$ in the MA gauge with $U(1)_3$ Landau-gauge fixing at $\beta = 0$ (thin line), 1.0 (dotted curve), 2.4 (solid curve), 3.0 (dashed curve).

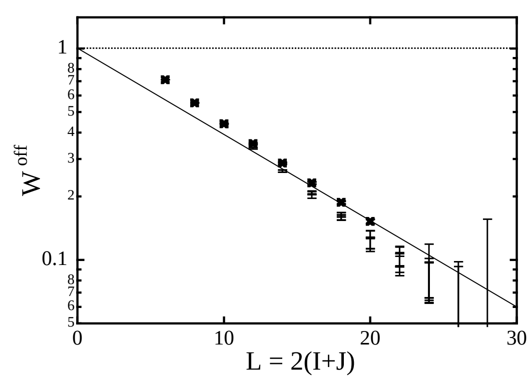

Fig.4 The comparison between the analytical estimation (straight line) and the lattice data $(\times)$ of the off-diagonal gluon contribution $W_C^{\rm off}$ for the Wilson loop as the function of $L \equiv 2(I+J)$ in the MA gauge at $\beta = 2.4$.

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